Online Appendix for
"Structural Scenario Analysis with SVARs"

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A Vector notation for VAR forecasts

Assume that we want to forecast the observables for some period ahead using the VAR in Equation (1) of the paper. Then, we can write

$$\mathbf{y}'_{t+h} = \mathbf{b}'_{t+h} + \sum_{j=1}^{h} \boldsymbol{\varepsilon}'_{t+j}\mathbf{M}_{h-j} \text{ for all } 1 < t < T \text{ and all } h > 0,$$

where

$$\begin{aligned}
&\mathbf{b}'_{t+h} = \mathbf{c}\mathbf{K}_{h-1} + \sum_{\ell=1}^{p} \mathbf{y}'_{t+1-\ell}\mathbf{N}_h^{\ell} \\
&\mathbf{K}_0 = \mathbf{I}_n \\
&\mathbf{K}_i = \mathbf{K}_0 + \sum_{j=1}^{i} \mathbf{K}_{i-j}\mathbf{B}_j \text{ if } i > 0 \\
&\mathbf{N}_1^{\ell} = \mathbf{B}_{\ell} \\
&\mathbf{N}_i^{\ell} = \sum_{j=1}^{i-1} \mathbf{N}_{i-j}^{\ell}\mathbf{B}_j + \mathbf{B}_{i+\ell-1} \text{ if } i > 1 \\
&\mathbf{M}_0 = \mathbf{A}_0^{-1} \\
&\mathbf{M}_i = \sum_{j=1}^{i} \mathbf{M}_{i-j}\mathbf{B}_j \text{ if } i > 0 \\
&\mathbf{B}_j = \mathbf{0}_{n\times n} \text{ if } j > p.
\end{aligned}$$

Then

$$\mathbf{y}'_{t+1,t+h} = \mathbf{b}'_{t+1,t+h} + \boldsymbol{\varepsilon}'_{t+1,t+h}\mathbf{M} \text{ for all } 1 < t < T \text{ and all } h > 0,$$

where $\mathbf{y}'_{t+1,t+h} = \left(\mathbf{y}'_{t+1} \dots \mathbf{y}'_{t+h}\right)$, $\mathbf{b}'_{t+1,t+h} = \left(\mathbf{b}'_{t+1} \dots \mathbf{b}'_{t+h}\right)$, $\boldsymbol{\varepsilon}'_{t+1,t+h} = \left(\boldsymbol{\varepsilon}'_{t+1} \dots \boldsymbol{\varepsilon}'_{t+h}\right)$, and

$$\mathbf{M} = \begin{pmatrix} \mathbf{M}_0 & \mathbf{M}_1 & \dots & \mathbf{M}_{h-1} \\ \mathbf{0} & \mathbf{M}_0 & \dots & \mathbf{M}_{h-2} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{M}_0 \end{pmatrix}.$$

It is easy to see that, given that $\mathbf{B} = \mathbf{A}_+\mathbf{A}_0^{-1}$, and $\boldsymbol{\Sigma} = (\mathbf{A}_0\mathbf{A}_0')^{-1}$, $\mathbf{M}'\mathbf{M}$ depends only on the reduced-form parameters, $\mathbf{B}$ and $\boldsymbol{\Sigma}$, even though $\mathbf{M}$ depends on the structural parameters, $\mathbf{A}_0$ and $\mathbf{A}_+$. The vector $\mathbf{b}'_{t+1,t+h}$ depends on the history of observables and the reduced-form parameters.

B Multiple Solutions for Conditional Forecasts

It is useful to examine what the general solution in Equations (10) and (12) imply for the three possible cases ($k < nh$, $k = nh$ and $k > nh$). When $k \leq nh$, the system of Equations (8) and (9) is consistent, and Equations (10) and (11) characterize the solution that minimizes the Frobenius norm of $\boldsymbol{\mu}_\varepsilon$ and $\boldsymbol{\Psi}_\varepsilon$.[1] Therefore, the Penrose solution is the one that envisages the smallest deviation of the mean and covariance matrix of $\tilde{\boldsymbol{\varepsilon}}_{T+1,T+h}$ from the mean and covariance matrix of $\boldsymbol{\varepsilon}_{T+1,T+h}$. In this case, $\mathbf{D}^\star = \mathbf{D}'\left(\mathbf{D}\mathbf{D}'\right)^{-1}$ and Equations (10) and (12) become[2]

$$\begin{aligned} \boldsymbol{\mu}_\varepsilon &= \mathbf{D}'\left(\mathbf{D}\mathbf{D}'\right)^{-1}\left(\mathbf{f}_{T+1,T+h} - \mathbf{C}\mathbf{b}_{T+1,T+h}\right) && \text{(B.1)}\\ \boldsymbol{\Sigma}_\varepsilon &= \mathbf{D}'\left(\mathbf{D}\mathbf{D}'\right)^{-1}\boldsymbol{\Omega}_f\left(\mathbf{D}\mathbf{D}'\right)^{-1}\mathbf{D} + \left(\mathbf{I}_{nh} - \mathbf{D}'\left(\mathbf{D}\mathbf{D}'\right)^{-1}\mathbf{D}\right). && \text{(B.2)} \end{aligned}$$

Clearly, if $k = nh$ we have that $\mathbf{D}^\star = \mathbf{D}^{-1}$ and Equations (10) and (12) become

$$\begin{aligned} \boldsymbol{\mu}_\varepsilon &= \mathbf{D}^{-1}\left(\mathbf{f}_{T+1,T+h} - \mathbf{C}\mathbf{b}_{T+1,T+h}\right)\\ \boldsymbol{\Sigma}_\varepsilon &= \mathbf{D}^{-1}\boldsymbol{\Omega}_f\left(\mathbf{D}^{-1}\right)'. \end{aligned}$$

When $k > nh$, the system is inconsistent, i.e., there is no solution to the system defined by Equations (8) and (9). In other words, not all the restrictions can be satisfied simultaneously. In this case Expressions (10) and (11) are best approximated solutions (see Penrose, 1956), meaning that they minimize

$$\left\|\mathbf{C}\mathbf{b}_{T+1,T+h} + \mathbf{D}\boldsymbol{\mu}_\varepsilon - \mathbf{f}_{T+1,T+h}\right\| \text{ and } \left\|\mathbf{D}\left(\mathbf{I}_{nh} + \boldsymbol{\Psi}_\varepsilon\right)\mathbf{D}' - \boldsymbol{\Omega}_f\right\|,$$

respectively, where we are using the Frobenius norm again.[3] In this case, $\mathbf{D}^\star = \left(\mathbf{D}'\mathbf{D}\right)^{-1}\mathbf{D}'$ and

[1] See Golub and Van Loan (1996), p. 55, for a definition of the Frobenius norm.

[2] It is worth noticing that Equations (10) and (12) are equivalent to the ones provided by Andersson et al. (2010), although they only consider the classic conditional forecasting exercise in which conditions are placed on observables. We thank an anonymous referee for suggesting to us this alternative and more straightforward derivation.

[3] For inconsistent systems the Penrose solution gives the same weight to all the constraints. If a researcher wanted to give different weight to the different restrictions, the solution can be easily amended (see Ben-israel and Greville, 2001, p.117).

Equations (10) and (12) become

$$\begin{aligned}\boldsymbol{\mu}_{\varepsilon} &= \left(\mathbf{D}'\mathbf{D}\right)^{-1}\mathbf{D}'\left(\mathbf{f}_{T+1,T+h} - \mathbf{C}\mathbf{b}_{T+1,T+h}\right) \\ \boldsymbol{\Sigma}_{\varepsilon} &= \left(\mathbf{D}'\mathbf{D}\right)^{-1}\mathbf{D}'\boldsymbol{\Omega}_{f}\mathbf{D}\left(\mathbf{D}'\mathbf{D}\right)^{-1}.\end{aligned}$$

C Proof of Proposition 1

In this Appendix we provide a proof for Proposition 1 in the paper. Let the following minimization problem be written

$$\min_{\boldsymbol{\mu},\boldsymbol{\Sigma}} D_{KL}\left(\mathcal{N}\left(\boldsymbol{\mu},\boldsymbol{\Sigma}\right)||\mathcal{N}_{UF}\right)$$

subject to

$$\mathbf{C}\boldsymbol{\mu} = \mathbf{f}_{T+1,T+h} \tag{C.1}$$

$$\mathbf{C}\boldsymbol{\Sigma}\mathbf{C}' = \boldsymbol{\Omega}_{f}. \tag{C.2}$$

For the multivariate Normal case $D_{KL}\left(\mathcal{N}\left(\boldsymbol{\mu},\boldsymbol{\Sigma}\right)||\mathcal{N}_{UF}\right)$ equals

$$\frac{1}{2}\left\{\mathtt{tr}\left[\left(\mathbf{M}'\mathbf{M}\right)^{-1}\boldsymbol{\Sigma}\right] + \left(\mathbf{b}_{T+1,T+h} - \boldsymbol{\mu}\right)'\left(\mathbf{M}'\mathbf{M}\right)^{-1}\left(\mathbf{b}_{T+1,T+h} - \boldsymbol{\mu}\right) - nh + \ln\left[\frac{\det\left(\mathbf{M}'\mathbf{M}\right)}{\det\left(\boldsymbol{\Sigma}\right)}\right]\right\}$$

Solution for the mean The solution for the mean of the conditional forecast solves the following FOCs

$$\mathbf{0}_{k\times 1} = \mathbf{C}\boldsymbol{\mu} - \mathbf{f}_{T+1,T+h}, \tag{C.3}$$

$$\mathbf{0}_{1\times nh} = \left(\mathbf{b}_{T+1,T+h} - \boldsymbol{\mu}\right)'\left(\mathbf{M}'\mathbf{M}\right)^{-1} - \boldsymbol{\lambda}\mathbf{C} \tag{C.4}$$

where $\boldsymbol{\lambda}$ is a $1 \times k$ vector of Lagrangian multipliers associated with the mean constraints in (C.1). Equation (C.4) implies that

$$\begin{aligned}\boldsymbol{\lambda}\mathbf{C} &= \left(\mathbf{b}_{T+1,T+h} - \boldsymbol{\mu}\right)'\left(\mathbf{M}'\mathbf{M}\right)^{-1} \\ \boldsymbol{\lambda}\mathbf{C}\mathbf{M}' &= \left(\mathbf{b}_{T+1,T+h} - \boldsymbol{\mu}\right)'\mathbf{M}^{-1} \\ \boldsymbol{\lambda}\mathbf{D} &= \left(\mathbf{b}_{T+1,T+h} - \boldsymbol{\mu}\right)'\mathbf{M}^{-1} \\ \mathbf{0}_{1\times nh} &= \left(\mathbf{b}_{T+1,T+h} - \boldsymbol{\mu}\right)'\mathbf{M}^{-1}\widehat{\mathbf{D}}\end{aligned}$$

where $\widehat{\mathbf{D}}$ denotes the $(nh \times nh)$ annihilator matrix such that $\mathbf{D}\widehat{\mathbf{D}} = \mathbf{0}_{k\times nh}$; therefore, $\widehat{\mathbf{D}} = \mathbf{I}_{nh} - \mathbf{D}'\left(\mathbf{D}\mathbf{D}'\right)^{-1}\mathbf{D} = \mathbf{I}_{nh} - \mathbf{D}^{\star}\mathbf{D}$, where $\mathbf{D}^{\star}$ denotes the Moore-Penrose inverse of $\mathbf{D}$ and has the following properties $\widehat{\mathbf{D}}' = \widehat{\mathbf{D}}$ (since $(\mathbf{D}^{\star}\mathbf{D})' = \mathbf{D}^{\star}\mathbf{D}$) and $\widehat{\mathbf{D}}^{-1} = \widehat{\mathbf{D}}$. Therefore

$$\widehat{\mathbf{D}}\left(\mathbf{M}'\right)^{-1}\boldsymbol{\mu} = \widehat{\mathbf{D}}\left(\mathbf{M}'\right)^{-1}\mathbf{b}_{T+1,T+h}$$

substituting for the definition of the annihilator matrix the previous equation implies that

$$\begin{aligned}
\left(\mathbf{M}'\right)^{-1}\boldsymbol{\mu} &= \left(\mathbf{M}'\right)^{-1}\mathbf{b}_{T+1,T+h} + \mathbf{D}^{\star}\mathbf{D}\left(\mathbf{M}'\right)^{-1}\left(\boldsymbol{\mu} - \mathbf{b}_{T+1,T+h}\right) \\
\boldsymbol{\mu} &= \mathbf{b}_{T+1,T+h} + \mathbf{M}'\mathbf{D}^{\star}\mathbf{D}\left(\mathbf{M}'\right)^{-1}\left(\boldsymbol{\mu} - \mathbf{b}_{T+1,T+h}\right) \\
&= \mathbf{b}_{T+1,T+h} + \mathbf{M}'\mathbf{D}^{\star}\left(\mathbf{C}\boldsymbol{\mu} - \mathbf{C}\mathbf{b}_{T+1,T+h}\right) \\
&= \mathbf{b}_{T+1,T+h} + \mathbf{M}'\mathbf{D}^{\star}\left(\mathbf{f}_{T+1,T+h} - \mathbf{C}\mathbf{b}_{T+1,T+h}\right)
\end{aligned}$$

which is equivalent to the solution for the mean of the conditional forecast in Equation (13) in Section 2 of the paper.

Solution for the variance The solution for the variance of the conditional forecast solves the following FOCs

$$\begin{aligned}
\mathbf{0}_{k\times k} &= \mathbf{C}\boldsymbol{\Sigma}\mathbf{C}' - \boldsymbol{\Omega}_f, && \text{(C.5)} \\
\mathbf{0}_{nh\times nh} &= \frac{1}{2}\frac{\partial \mathtt{tr}\left[\left(\mathbf{M}'\mathbf{M}\right)^{-1}\boldsymbol{\Sigma}\right]}{\partial\boldsymbol{\Sigma}} - \frac{1}{2}\frac{\partial \ln\left[\det\left(\boldsymbol{\Sigma}\right)\right]}{\partial\boldsymbol{\Sigma}} + \frac{\partial \mathtt{tr}\left[\mathbf{S}\left(\mathbf{C}\boldsymbol{\Sigma}\mathbf{C}' - \boldsymbol{\Omega}_f\right)\right]}{\partial\boldsymbol{\Sigma}}. && \text{(C.6)}
\end{aligned}$$

where $\mathbf{S}$ is a square positive semidefinite matrix of Lagrangian multipliers associated with the variance constraints in (C.2). Equation (C.6) implies that

$$\begin{aligned}
\left(\mathbf{M}'\mathbf{M}\right)^{-1} - \boldsymbol{\Sigma}^{-1} + 2\mathbf{C}'\mathbf{S}'\mathbf{C} &= \mathbf{0}_{nh\times nh} \\
\mathbf{I}_{nh} - \mathbf{M}\boldsymbol{\Sigma}^{-1}\mathbf{M}' + 2\mathbf{D}'\mathbf{S}'\mathbf{D} &= \mathbf{0}_{nh\times nh}
\end{aligned}$$

where we have made use of the definition of $\mathbf{D}$ $(= \mathbf{C}\mathbf{M}')$, therefore

$$\mathbf{D}'\mathbf{S}'\mathbf{D} = \frac{1}{2}\left(\mathbf{M}\boldsymbol{\Sigma}^{-1}\mathbf{M}' - \mathbf{I}_{nh}\right) \tag{C.7}$$

Equation (C.7) together with Equation (C.5) implies that

$$\begin{aligned}
\mathbf{D}'\mathbf{S}'\mathbf{D} &= \frac{1}{2}\left(\mathbf{M}\boldsymbol{\Sigma}^{-1}\mathbf{M}' - \mathbf{I}_{nh}\right) \\
\mathbf{S}' &= \frac{1}{2}\mathbf{D}^{\star\prime}\left(\mathbf{M}\boldsymbol{\Sigma}^{-1}\mathbf{M}' - \mathbf{I}_{nh}\right)\mathbf{D}^{\star} \\
&= \frac{1}{2}\mathbf{D}^{\star\prime}\left[\mathbf{M}\left(\mathbf{C}^{*}\boldsymbol{\Omega}_f\mathbf{C}^{*\prime}\right)^{-1}\mathbf{M}' - \mathbf{I}_{nh}\right]\mathbf{D}^{\star} \\
&= \frac{1}{2}\mathbf{D}^{\star\prime}\left[\mathbf{M}\mathbf{C}'\boldsymbol{\Omega}_f^{-1}\mathbf{C}\mathbf{M}' - \mathbf{I}_{nh}\right]\mathbf{D}^{\star} \\
&= \frac{1}{2}\mathbf{D}^{\star\prime}\left[\mathbf{D}'\boldsymbol{\Omega}_f^{-1}\mathbf{D}' - \mathbf{I}_{nh}\right]\mathbf{D}^{\star} \\
&= \frac{1}{2}(\boldsymbol{\Omega}_f^{-1} - \mathbf{D}^{\star\prime}\mathbf{D}^{\star})
\end{aligned}$$

therefore we can solve for the variance under the conditional forecast from Equation (C.7)

$$\begin{aligned}
\mathbf{M}\boldsymbol{\Sigma}^{-1}\mathbf{M}' &= \mathbf{I}_{nh} + \mathbf{D}'(\boldsymbol{\Omega}_f^{-1} - \mathbf{D}^{\star\prime}\mathbf{D}^{\star})\mathbf{D} \\
\mathbf{M}\boldsymbol{\Sigma}^{-1}\mathbf{M}' &= \mathbf{D}'\boldsymbol{\Omega}_f^{-1}\mathbf{D} + (\mathbf{I}_{nh} - \mathbf{D}'\mathbf{D}^{\star\prime}\mathbf{D}^{\star}\mathbf{D}) \\
\boldsymbol{\Sigma}^{-1} &= \mathbf{M}^{-1}(\mathbf{D}'\boldsymbol{\Omega}_f^{-1}\mathbf{D} + \widehat{\mathbf{D}})\left(\mathbf{M}'\right)^{-1}
\end{aligned}$$

which implies that

$$\begin{aligned}
\boldsymbol{\Sigma} &= \mathbf{M}'(\mathbf{D}'\boldsymbol{\Omega}_f^{-1}\mathbf{D} + \widehat{\mathbf{D}})^{-1}\mathbf{M} \\
&= \mathbf{M}'(\mathbf{D}^{\star}\boldsymbol{\Omega}_f\mathbf{D}^{\star\prime} + \widehat{\mathbf{D}})\mathbf{M}
\end{aligned}$$

and is equivalent to the solution for the conditional forecast in Equation (14) in Section 2 of the paper.[4]

D Details of the algorithm

In this section we develop algorithms to implement structural scenario analysis. Conditional-on-variables and conditional-on-shocks forecasts can be implemented as special cases of the former. The algorithms we present can be easily extended to any identification scheme provided that an

[4]To solve for the variance we have made use of the property that given two square matrices $\mathbf{A}$ and $\mathbf{B}$, $(\mathbf{A}+\mathbf{B})^{-1} = (\mathbf{A}^{-1} + \mathbf{B}^{-1})$ if $\mathbf{A}\mathbf{B}^{-1}\mathbf{A} = \mathbf{B}\mathbf{A}^{-1}\mathbf{B}$ (see, e.g., Searle, 1982). The latter conditions in our case are verified: $\mathbf{D}'\Omega_f^{-1}\mathbf{D}\widehat{\mathbf{D}}^{-1}\mathbf{D}'\Omega_f^{-1}\mathbf{D} = \widehat{\mathbf{D}}\mathbf{D}^{\star\prime}\Omega_f\mathbf{D}^{\star}\widehat{\mathbf{D}} = \mathbf{0}_{nh\times nh}$, since from the definition of the annihilator matrix $\mathbf{D}\widehat{\mathbf{D}} = \mathbf{0},$ and $\widehat{\mathbf{D}} = \widehat{\mathbf{D}}'$ and $\widehat{\mathbf{D}}^{-1} = \widehat{\mathbf{D}}$.

algorithm is available to draw from the structural parameters. To simplify both the notation and the exposition, when presenting the algorithm we focus on sign restrictions. Specifically, we extend the Gibbs sampler algorithm in Waggoner and Zha (1999) to implement the structural scenario analysis in set and partially identified SVARs.

In particular, we assume that if $(\mathbf{A}_0, \mathbf{A}_+)$ satisfy the s sign restrictions, then

$$\mathbf{F}(\mathbf{A}_0, \mathbf{A}_+) > \mathbf{0}_{s\times 1}.$$

Before describing the algorithm, it should be clear that our objective is to draw from the following joint posterior

$$p(\tilde{\mathbf{y}}_{T+1,T+h}, \mathbf{A}_0, \mathbf{A}_+ | \mathbf{y}^T, \mathbf{IR}\left(\mathbf{A}_0, \mathbf{A}_+\right), \boldsymbol{R}\left(\tilde{\mathbf{y}}_{T+1,T+h}, \mathbf{A}_0, \mathbf{A}_+\right)), \tag{D.1}$$

where $\mathbf{IR}\left(\mathbf{A}_0, \mathbf{A}_+\right) = \{(\mathbf{A}_0, \mathbf{A}_+) : \mathbf{F}(\mathbf{A}_0, \mathbf{A}_+) > \mathbf{0}_{s\times 1}\}$ are the identification restrictions and $\mathbf{R}(\tilde{\mathbf{y}}_{T+1,T+h}, \mathbf{A}_0, \mathbf{A}_+) = \{\tilde{\mathbf{y}}_{T+1,T+h} : \hat{\mathbf{C}}\tilde{\mathbf{y}}_{T+1,T+h} \sim \mathcal{N}\left(\hat{\mathbf{f}}_{T+1,T+h}, \hat{\boldsymbol{\Omega}}_f\right)\}$ are the restrictions associated with the structural scenario. Note that $\mathbf{R}(\tilde{\mathbf{y}}_{T+1,T+h}, \mathbf{A}_0, \mathbf{A}_+)$ depends on $(\mathbf{A}_0, \mathbf{A}_+)$ because $\hat{\mathbf{f}}_{T+1,T+h}$, $\hat{\mathbf{C}}$, and $\hat{\boldsymbol{\Omega}}_f$ depend on $(\mathbf{A}_0, \mathbf{A}_+)$.

As explained by Waggoner and Zha (1999), if one wants to take into account identification and parameter uncertainty, as well as the uncertainty associated with the future path of the shocks and, possibly, the conditional path of the restricted forecast, drawing the posterior distribution described by Equation (D.1) becomes a challenging task. It is tempting, in a first step to, draw the structural parameters from their distribution conditional on $\mathbf{y}^T$ and the identification restrictions, and in a second step to draw $\tilde{\mathbf{y}}_{T+1,T+h}$ conditional on $\mathbf{y}^T$, the restrictions in Equation (15) of the paper, and the structural parameters, using Equations (13) and (14) of the paper. However, this procedure ignores the restrictions in Equation (15) when drawing the structural parameters and, hence, would not lead to a draw from the desired joint posterior described in Equation (D.1). Instead, to draw from the joint distribution of interest, a Gibbs sampler procedure can be constructed that iterates between draws from the described conditional distributions of the structural parameters and $\tilde{\mathbf{y}}_{T+1,T+h}$.[5]

In this appendix we present a particular implementation of Algorithm 1 that follows Faust

[5] The algorithm below is an extension of the one developed by Waggoner and Zha (1999), with the key difference being that we now need to identify the SVAR.

(1998), Uhlig (2005), and Canova and Nicolo (2002), and Rubio-Ramirez et al. (2010) and draw from a conjugate uniform-normal-inverse-Wishart posterior over the orthogonal reduced-form parameterization $(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q})$ and transform the draws into the structural parameterization $(\mathbf{A}_0, \mathbf{A}_+)$. Arias et al. (2018) show that this transformation induces a normal-generalized-normal posterior over the structural parameterization, and highlight the useful properties of using conjugate prior distributions. On the contrary, Baumeister and Hamilton (2015) prefer to use priors stated directly over the structural parameterization, $(\mathbf{A}_0, \mathbf{A}_+)$. Their argument is that since the implementation of Arias et al. (2018) amounts to an implicit and informative prior over the objects of interest to researchers, it is better to define it directly. Baumeister and Hamilton's (2015) approach has an advantage when at least some, if not all, the parameters have an economic interpretation, but could be computationally challenging, especially in large systems. If one wanted to follow that route, our algorithm could be easily modified to draw directly from $(\mathbf{A}_0, \mathbf{A}_+)$.

Algorithm 1. *Initialize* $\mathbf{y}^{T+h,(0)} = [\mathbf{y}^T, \tilde{\mathbf{y}}^{(0)}_{T+1,T+h}]$.

1. *Conditioning on* $\mathbf{y}^{T+h,(i-1)} = [\mathbf{y}^T, \tilde{\mathbf{y}}^{(i-1)}_{T+1,T+h}]$*, draw* $(\mathbf{B}^{(i)}, \mathbf{\Sigma}^{(i)})$ *from the posterior distribution of the reduced-form parameters.*

2. *Draw* $\mathbf{Q}^{(i)}$ *independently from the uniform distribution over the set of orthogonal matrices.*

3. *Keep* $(\mathbf{B}^{(i)}, \mathbf{\Sigma}^{(i)}, \mathbf{Q}^{(i)})$ *if* $\mathbf{F}(f_h^{-1}(\mathbf{B}^{(i)}, \mathbf{\Sigma}^{(i)}, \mathbf{Q}^{(i)})) > \mathbf{0}_{s\times 1}$*, otherwise return to Step 1.*

4. *Conditioning on* $(\mathbf{A}_0^{(i)}, \mathbf{A}_+^{(i)}) = f_h(\mathbf{B}^{(i)}, \mathbf{\Sigma}^{(i)}, \mathbf{Q}^{(i)})$ *and* $\mathbf{y}^T$*, draw* $\tilde{\mathbf{y}}^{(i)}_{T+1,T+h} \sim \mathcal{N}(\boldsymbol{\mu}_y, \mathbf{\Sigma}_y)$ *using Equations (10) and (12) of the paper.*

5. *Return to Step 1 until the required number of draws has been obtained.*

The natural initialization of $\tilde{\mathbf{y}}^{(0)}_{T+1,T+h}$ can be done at the mean of the unconditional forecast or a random draw from its posterior. Steps 1 and 2 draw from posterior uniform-normal-inverse-Wishart over the orthogonal reduced-form parameterization conditional on $\mathbf{y}^{T+h,(i-1)}$. Step 3 checks the sign restrictions, while Step 4 draws $\tilde{\mathbf{y}}^{(i)}_{T+1,T+h} \sim \mathcal{N}(\boldsymbol{\mu}_y, \mathbf{\Sigma}_y)$ using Equations (10) and (12) of the paper. It is important to notice that because Steps 1 and 2 are conditional on $\tilde{\mathbf{y}}^{T+h,(i-1)}$, the restriction in Equation (15) is taken into account when drawing the structural parameters.

Clearly Steps 1-3 are an accept-reject algorithm to draw from the posterior of $(\mathbf{A}_0^{(i)}, \mathbf{A}_+^{(i)})$ conditional on the identification restrictions, as in Rubio-Ramirez et al. (2010), but taking into account the restrictions on $\mathbf{y}^{T+h,(i-1)}$. Step 4 draws $\tilde{\mathbf{y}}^{(i)}_{T+1,T+h}$ conditional on $(\mathbf{A}_0^{(i)}, \mathbf{A}_+^{(i)})$ using

Equation (15). For the case in which the model is exactly identified, such as in the traditional recursive ordering using the Cholesky decomposition (see Kilian and Lutkepohl, 2017), the algorithm is trivially modified by choosing a function h that reflects the restrictions, setting $\mathbf{Q} = \mathbf{I}_n$ in Step 2 and skipping Step 3. The algorithm can also be extended to implement the recently proposed narrative sign restrictions as in Antolin-Diaz and Rubio-Ramirez (2018) and zero restrictions by the methods described in Arias et al. (2018). In both cases, our algorithm requires an importance-sampling step to draw from the posterior of $(\mathbf{A}_0, \mathbf{A}_+)$, which needs to be done after Step 3. This approach is known as Gibbs sampling by sampling-importance-resampling as described in Koch (2007). In particular, the resampling step requires taking multiple draws of the triplet $(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q})$ by repeating Steps 1-3 above, which are then re-weighted using importance weights as described in Antolin-Diaz and Rubio-Ramirez (2018) and Arias et al. (2018) respectively. Finally, a random draw from the reweighted triplets $(\mathbf{B}, \mathbf{\Sigma}, \mathbf{Q})$ is selected before proceeding to Step 4.

E Alternative implementation using Kalman filtering

Clarida and Coyle (1984) show that one can implement the Doan et al. (1986) conditional point forecasts using the Kalman filter. Building on this insight, Banbura et al. (2015) suggest that state-space methods can also be used to obtain conditional-on-observables forecasts as in Section 2. We now show that this equivalent algorithm using the Kalman filter can also be applicable to the structural scenario analysis as well. As we will see, however, the Kalman filter implementation will be in most cases computationally less efficient than the general, closed-form solutions provided in the paper. Moreover, it does not easily lend itself to implement more flexible linear restrictions such as the "Back to the Price Level Target" exercise described in Section 5.

The state-space representation

We begin by outlining the state-space representation that allows the use of the Kalman filter to compute conditional forecasts. Consider the following state-space form consistent with the model in Section 2:

$$\mathbf{z}_t' = \mathbf{D} + \mathbf{z}_{t-1}'\mathbf{F} + \varepsilon_t'\mathbf{G} \tag{E.1}$$

$$\tilde{\mathbf{y}}_t' = \mathbf{H}\mathbf{z}_t' \tag{E.2}$$

The state vector $\mathbf{z}_t' = [\mathbf{y}_t', \ldots, \mathbf{y}_{t-p+1}', \boldsymbol{\varepsilon}_t']$ stacks the endogenous variables, their lags, and the exogenous shocks, where $\mathbf{y}_t$ is an $n \times 1$ vector of variables at time t, and $\boldsymbol{\varepsilon}_t$ is an $n \times 1$ vector of structural shocks at time t. The measurement vector $\tilde{\mathbf{y}}_t' = [\mathbf{y}_t', \boldsymbol{\varepsilon}_t']$ stacks the endogenous variables and the exogenous shocks. As we will see below, since the structural shocks are unobserved, the measurement vector will contain missing observations corresponding to the entries of $\boldsymbol{\varepsilon}_t$. The system matrices $\mathbf{D}$, $\mathbf{F}$, $\mathbf{G}$, and $\mathbf{H}$ are constructed from the SVAR parameters as shown below:

$$\mathbf{D} = \begin{bmatrix} \mathbf{c} \\ \underset{n \cdot p \times 1}{\mathbf{0}} \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \underset{n \cdot p \times n \cdot p}{\tilde{\mathbf{B}}} & \underset{n \cdot p \times n}{\mathbf{0}} \\ \underset{n \times n \cdot p}{\mathbf{0}} & \underset{n \times n}{\mathbf{0}} \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} h(\boldsymbol{\Sigma})\mathbf{Q}^{-1} \\ \underset{n \cdot (p-1) \times n}{\mathbf{0}} \\ \underset{n \times n}{\mathbf{I}} \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} \underset{n \times n}{\mathbf{I}} & \underset{n \times n \cdot p}{\mathbf{0}} \\ \underset{n \times n \cdot p}{\mathbf{0}} & \underset{n \times n}{\mathbf{I}} \end{bmatrix},$$

where $\tilde{\mathbf{B}}$ is known as the companion matrix of the VAR and is defined as

$$\tilde{\mathbf{B}} = \begin{bmatrix} \mathbf{B}_1 & \mathbf{B}_2 & & \ldots & \mathbf{B}_p \\ \mathbf{I} & \mathbf{0} & & & \\ \mathbf{0} & \mathbf{I} & \ddots & & \vdots \\ \vdots & & & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \ldots & \ldots & \mathbf{0} & \mathbf{I} \end{bmatrix}.$$

The SVAR representation in Equation 1 of the paper and the state-space representation in Equations (E.1)-(E.2) described above are equivalent, and both can be used to compute conditional forecasts. With the model in state-space one can use the simulation smoother (see, e.g., Durbin and Koopman, 2012) to generate a draw of the state vector $\tilde{\mathbf{y}}_t'$, conditional on the observations and the parameters of the model. Therefore, as outlined by Banbura et al. (2015) for the reduced-form case, one can treat the future path of conditioning variables as observed, while leaving the future path of the remaining variables as missing observations.[6]

The same procedure can be adapted for the case of the structural scenario. Suppose that one

[6] To allow for missing observations, the standard Kalman filtering recursions must be modified. For this purpose, a number of equivalent solutions have been proposed (see Durbin and Koopman (2012), p. 112, for a textbook treatment). Here we follow the approach of Mariano and Murasawa (2003) and Camacho and Perez-Quiros (2010), which essentially impose the condition that the Kalman gain associated with the missing observation is mechanically zero. This implies that the Kalman filter "skips" missing observations, effectively marginalizing the likelihood with respect to the missing observations (see Brockwell and Davis (1991), Section 12.3, and Brockwell et al. (1990)).

wants to produce forecasts for h time periods into the future. Denote with $\star$ an observation for which all data entries are missing, denote $(\hat{y}_{i,t}, \star)'$ for $T+1 \leq t \leq T+h$ a post-sample observation in which the value of endogenous variable i is assumed to be known and fixed, the rest being missing data, and denote $(\hat{\varepsilon}_{j,t}, \star)'$ for $T+1 \leq t \leq T+h$ a post-sample observation in which the value of structural shocks j is constrained (for instance to reflect its unconditional standard normal distribution), the observation for other shocks being missing data. All of the cases mentioned above can be considered special cases of the state-space representation (E.1)-(E.2), in which the measurement vector is constructed as $\tilde{\mathbf{y}}_t'$ in the following way:

$$\tilde{\mathbf{y}}_t' = [\mathbf{y}_t', *], \qquad \text{for } 1 \leq t \leq T, \tag{E.3}$$

$$\tilde{\mathbf{y}}_t' = [(\hat{y}_{i,t}, *)', (*, \theta_t)'], \qquad \text{for } T+1 \leq t \leq T+h, \tag{E.4}$$

where θ_t denotes the vector of constrained structural shocks and is a $1 \times n-1$ draw from the unconditional distribution of ε_t, i.e., from an independent standard normal distribution. In other words, n columns and h rows of missing observations are appended to the data set containing the endogenous variables, the known future path for variable i for periods $T+1$ to $T+h$ is used to fill the corresponding missing observations, and the missing observations for the structural shocks *other than* the shock of interest are filled in with draws from their unconditional distribution.[7] In Step 4 of Algorithm 1, one can then draw $y^{(i)}_{T+1,T+h}$ using the Kalman smoother, as opposed to a multivariate Gaussian distribution with mean and variance given by Equations (10)-(12) of the paper. These two procedures produce identical results.

Computational efficiency

Since the Kalman filtering-based procedure provides results identical to those of the algorithm outlined in the paper, we now assess which of the two methods is more computationally efficient. Equations (10)-(12) of the paper involve operations of high dimensional matrix objects which modern computing langues are well suited to compute efficiently, whereas the procedure based on the Kalman filter works recursively leading to a higher computational burden.

Table E.1 investigates which of the two procedures is more efficient numerically for different

[7] If one is interested in computing only point forecasts, filling them with zeros, the unconditional mean, is appropriate.

Table E.1: Computational efficiency of conditional forecast methods

		$p=1$		$p=6$		$p=12$		$p=24$	
	Horizon	Baseline	KF	Baseline	KF	Baseline	KF	Baseline	KF
n=3	h=1	**0.49**	27.37	**0.5**	34.68	**0.52**	51.77	**0.56**	88.15
	h=6	**0.8**	28.09	**1.12**	35.32	**2.25**	49.02	**2.11**	90.35
	h=12	**1.29**	28.48	**3.1**	36.17	**5.53**	50.24	**5.42**	95.68
	h=24	**2.78**	29.76	**5.06**	37.92	**8.89**	52.72	**20.28**	98.19
	h=60	**12.37**	33.61	**18.51**	43.38	**31.35**	59.67	**69.64**	113.28
n=7	h=1	**0.53**	35.57	**0.55**	66.81	**0.51**	141.74	**0.61**	368.3
	h=6	**1.42**	36.21	**2.47**	67.34	**3.07**	139.76	**3.03**	375.16
	h=12	**2.81**	37.17	**4**	69.25	**5.43**	147.83	**7.79**	386.21
	h=24	**9.16**	38.68	**11.81**	72.16	**16.72**	148.68	**28.05**	403.88
	h=60	77.07	**43.67**	**83.9**	84.62	**102.7**	174.42	**150.89**	458.84
n=11	h=1	**0.58**	34.04	**0.61**	93.49	**0.55**	200.4	**0.68**	697.13
	h=6	**2.12**	35.06	**2.58**	91.7	**3**	209.97	**3.82**	738.55
	h=12	**5.61**	36.02	**7.6**	94.64	**9.97**	218.71	**11.62**	751.22
	h=24	**25.31**	38.38	**25.36**	99.79	**31.52**	231.55	**48.95**	821.48
	h=60	210.07	**44.63**	216.67	**126.62**	**247.11**	277.71	**308.01**	964.74
n=27	h=1	**0.9**	131.52	**0.65**	579.93	**1.28**	2241.43	**0.99**	12193.3
	h=6	**8.98**	130.98	**9.17**	622.57	**10.34**	2314.67	**12.88**	12335.05
	h=12	**35.94**	135.65	**40.92**	599.76	**45.23**	2351.54	**52.18**	12644.66
	h=24	224.8	**135.44**	**224.38**	625.57	**248.64**	2461.27	**285.89**	13288.57
	h=60	6301.38	**158.01**	3874.75	**732.05**	3980.7	**2857.22**	**4089.79**	15290.47

Note: The numbers denote the average time, in seconds (over 100 iterations), taken to calculate 1000 draws of conditional forecasts. n denotes the number of variables in the VAR, h denotes the forecast horizon and p denotes the number of lags in the VAR. The time of the fastest algorithm in each case is marked in boldface. These computations were performed in Matlab 2015a on an HP Z230 SFF workstation with an Intel Core i7-4770 CPU 3.40GHz processor and 7.74GB of usable RAM.

hypothetical problem settings.[8] The baseline implementation of Algorithm 1 is faster in the majority of cases, with the gains particularly large whenever the number of variables n or lags p is large. Only whenever h increases a lot relative to the number of variables and lags is small, drawing the conditional forecast using the Kalman smoother is more efficient.

F Details on the applications

Application 1: Monetary policy alternatives

Data

We work with the data set of Smets and Wouters (2007), which contains seven key US macroeconomic time series at the quarterly frequency: real output, consumption, investment, wages and hours

[8] We use deJong's (1988) implementation of the Kalman filter, as Banbura et al. (2015) show that this implementation is computationally faster than alternative implementations.

worked per capita, inflation as measured by the GDP deflator and the federal funds rate.[9] Their data set starts in Q1 1966, and we update it through Q4 2019. For the comparison with the DSGE model in Section 5, we use the original dataset in Smets and Wouters, downloaded from the AER website. We verified that the original dataset corresponds to the vintage available in March 2005. We make one modification to the dataset, which is to substitute the GDP deflator for the PCE deflator excluding food and energy as an indicator for inflation. We use the data for the March 2005 vintage. The motivation for this change is to be consistent with the fact that the Federal Reserve's inflation objective is stated in terms of the PCE deflator, and in particular many of its communications and internal documents refer to the less volatile ex-food and energy measure as an more reliable indicator of future inflation tendencies. Over the sample considered, the core PCE and the overal GDP deflator are 93% correlated.

For the applications updated through 2019, we udpate the dataset using data from Haver Analytics and using the corresponding tickers:

1. Gross Domestic Product (GDV@USECON)
2. Gross Domestic Product - Implicit Price Deflator (JGDP@USECON)
3. Core PCE Deflator (LXEHL@USECON)
4. Personal Consumption Expenditures, Non-Durable Goods and Services (CS@USECON + CN@USECON)
5. Fixed Private Investment + Consumer Durables (F@USECON + CD@USECON)
6. Total Economy Hours Worked (LXEHL@USECON)
7. Compensation of Employees (YCOMP@USECON)
8. Civilian Non-Institutional Population (LNN@USECON)
9. Federal Funds Rate (FFED@USECON)

Output, Consumption and Investment are divided by the GDP deflator and by population to obtain real, per-capita series. Hours are divided by population. Compensation is divided by hours worked and the GDP deflator to obtain real compensation per hour. To construct the measure of expect

[9]See the data appendix in Smets and Wouters (2007) for exact definitions of the variables.

interest rates we use series from the Philadelphia Fed Survey of Professional Forecasters 3 month T-bill expecations:

1. Current Quarter (ASAFC3T@SURVEYS)
2. 1 Quarter Ahead (ASAF13T@SURVEYS)
3. 2 Quarter Ahead (ASAF23T@SURVEYS)
4. 3 Quarter Ahead (ASAF33T@SURVEYS)
5. 4 Quarter Ahead (ASAF43T@SURVEYS)

Then, the variable

$$S_{t,h} \equiv E_t^{SPF}[i_{t+h}] - E_{t-1}^{SPF}[i_{t+h}] \tag{F.1}$$

for $h = 0, \dots, 3$ measures the *surprise* to expectations, i.e., the change in the information set, from $t-1$ to t. Campbell et al. (2012) point out that surprises across horizons h exhibit a strong factor structure, so we take the average across horizons, $\bar{S}_t \equiv 1/4 \sum_{h=0}^{3} S_{t,h}$ as a measure of the surprises to the average expected interest rate for the next year.

Estimation settings

We estimate a VAR with $p = 4$ lags using Bayesian methods and an informative "Minnesota" prior (Doan et al., 1986; Sims, 1993) over the reduced-form parameters. Also see Karlsson (2013) In order to deal with the absence of data for the survey in the early part of the sample, we need to add an additional step to Algorithm 1. We cast the model in state-space and, conditional on the previous draw of the reduced-form parameters, we draw the missing observations using a simulation smoother (e.g., Durbin and Koopman, 2012)

Application 2: Stress-testing

Data

In this section we report the definition and Haver mnemonics for the data used in Application 2, which are appended to the dataset used in Application 1:

- 3-month Treasury bill rate (FTBS3@USECON)

- unemployment rate (LR@USECON)
- 10-year yield (FCM10@USECON)
- S&P 500 index (SP500@USECON)
- house price index (USRSNHPM@USECON)
- oil price deflated by the core PCE deflator (PZTEXP@USECON/JCXFE@USECON)
- credit spread is the BAA credit spread minus the treasury long-term composite yield (FBAA@USECON-FLTG@USECON)
- TED spread (C111FRED@OECDMEI - FTBS3@USECON)
- ROE for all FDIC Insured Institutions (USARQ@FDIC).

The data are quarterly from 1966 to 2019.

Estimation settings

The VAR is estimated with gour lags and the Minnesota prior implemented using dummy observations (see Del Negro (2011) for details) over the reduced-form parameters.

G Anticipated Policy Shocks in the Smets-Wouters model

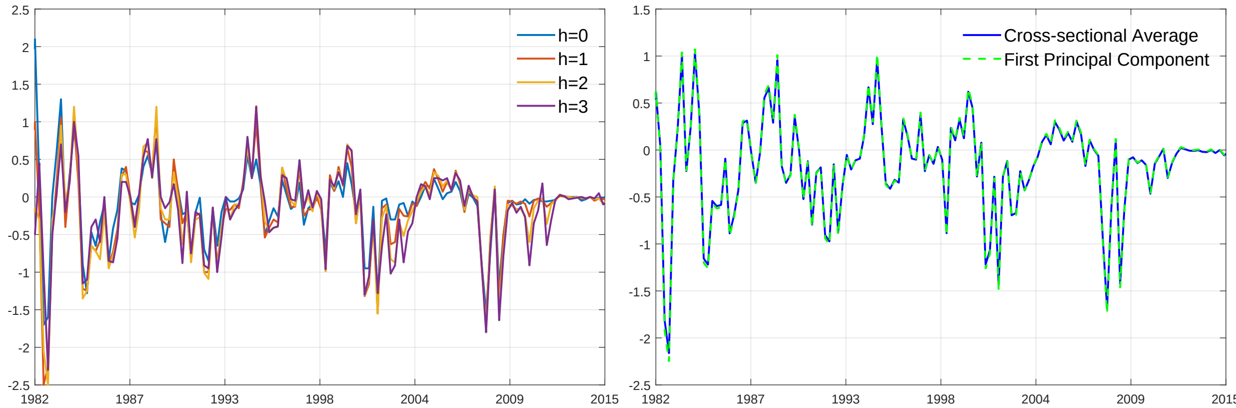
In the empirical section we compare the results from the VAR to the ones from a modified version of the Smets and Wouters (SW) model. In this appendix we describe the main modifications of the model and explain the use we make of it, in particular to inform us on the restrictions to impose for the identification of the anticipated monetary policy shock and on the feasibility of the identification of the structural shocks from the VAR.

The only modification of the model is the introduction of anticipated monetary policy shocks. In particular, modify the policy rule in the original model and introduce news shocks as follows

$$r_t = \rho r_{t-1} + (1-\rho)[r_\pi \pi_t + r_Y(y_t - y_t^p)] + \rho_{\Delta y}[(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + \epsilon_t^r + \sum_{j=0}^{3} v_{t-j,j}^r \quad \text{(G.1)}$$

where $v_{t-j,j}^r$ is an anticipated monetary policy shock, which corresponds to deviations from the normal conduct of policy revealed at time $t-j$ yet affecting the interest rate with a delay of j

Figure G.1: Federal Fund Surprises



Note. The left chart reports $S_{t,h} \equiv E_t^{SPF}[i_{t+h}] - E_{t-1}^{SPF}[i_{t+h}]$, for $h = 0, \ldots, 3$. The right panel of the figure contrasts the cross sectional average of the surprises with their first principal component.

quarters. In order to capture that idea that forward guidance announcements affect the path of the interest rate over multiple quarters, we assume that news shocks follow a simple MA(4) structure:

$$v_{t-j,j}^r = \eta_{t-j}^r \quad j = 0, \ldots, 3 \tag{G.2}$$

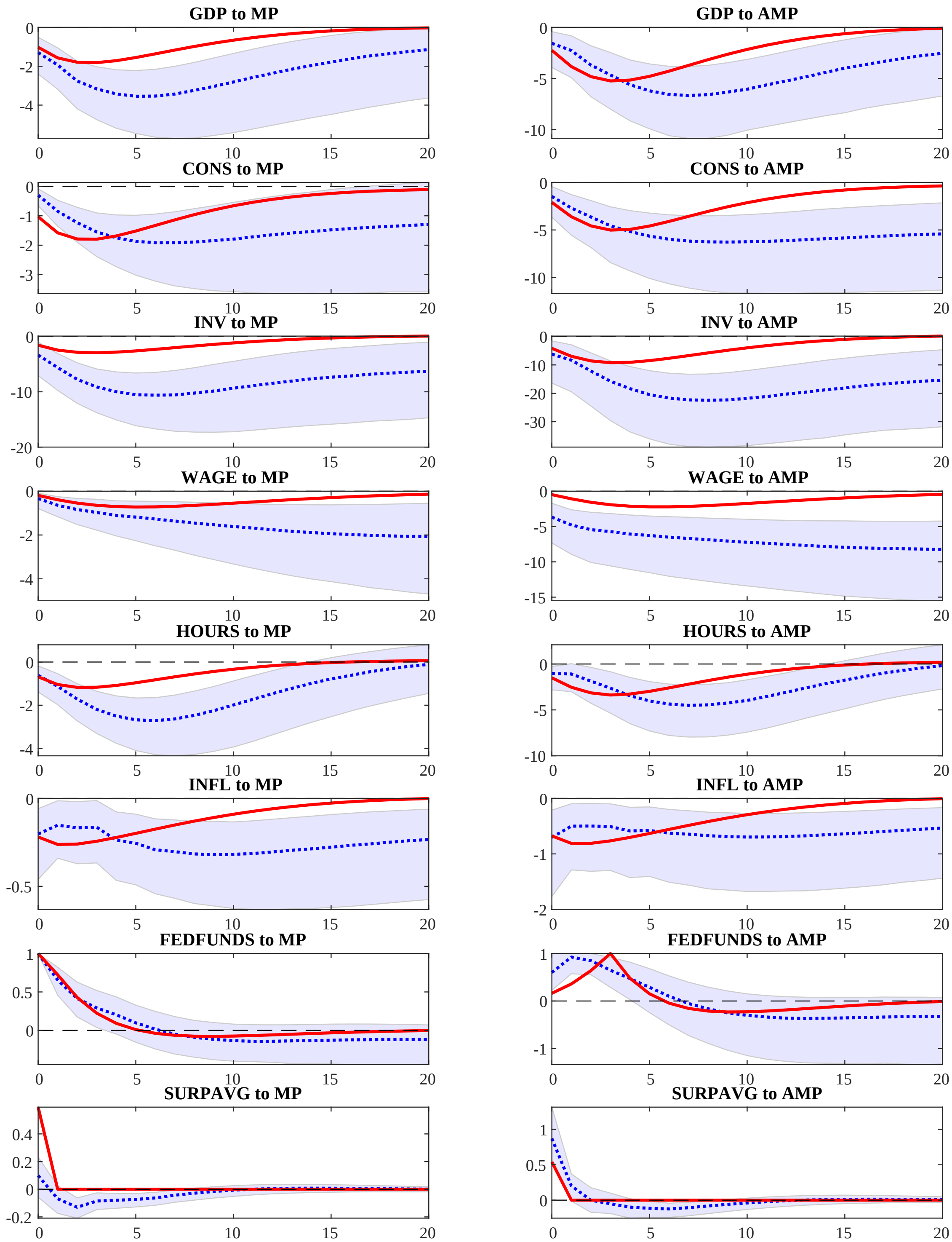
The way we enter anticipated monetary policy shocks into the model is in line with Campbell et al. (2012). The simplifying assumption of a single factor structure in equation G.2 reflects the observations that revisions in interest rate forecast at different quarters display a strong comovement and roughly equal volatility, as highlighted in Figure G.1. Therefore the key difference between a standard (unanticipated) monetary policy shock and a 'forward' guidance shock is that the latter is expected to affect the path of the interest rate for prolonged periods of time. In order to stay as close possible to the original SW model we assume that the law of motion of η_t^r follows an AR(1) and fix the parameters of this to be the same as the one of the unanticipated monetary policy shock.

The first question that we ask from the model is whether a seven variables VAR – featuring the original variables in the SW model – would able to recover an anticipated monetary policy shock, i.e. more specifically whether the VAR is informationally sufficient or fundamental. Fernández-Villaverde et al. (2007) put forward a simple metric, the 'poor man invertibility condition' (PMIC), which we can check on the implied VAR solution in the theoretical model. Fixing the parameters estimates to the modal estimate in SW, the PMIC shows that a 7 variables VAR is not fundamental in the structural shocks. However, expanding the model so as to include the average forecast revisions in the short term interest rate suggests that a VAR augmented with a surprise variable calculated as described in the text satisfies the invertibility requirement and is therefore informationally sufficient. This is computed in line with the definition given in Section 5 of the paper (equation (F.1) above) and with expectations computed under the assumption of rationality. However, we do not need the survey expectations to be fully rational in order for the VAR to be fundamental and identification to work.

We also use the model to derive plausible restrictions for the identification of the SVAR. Simulations from the model suggest that an anticipated monetary policy shock is associated with a pattern of signs of the IRFs which is consistent with the one of a traditional monetary policy shock. An anticipated monetary policy shock is also associated with a movement in the interest rate surprise variable, $\bar{S}_t$ that goes in the same direction than the shock. When comparing the impact of an unanticipated and an anticipated MP shock it is clear that, for a shock normalized to have the same initial impact on the short term interest rate, the anticipated monetary policy shocks are associated with larger average revisions in $\bar{S}_t$ than in the unanticipated case.[10] Figure G.2 compares the impulse response function of the SVAR and the SW model for a monetary policy (MP) shock and the anticipated monetary policy (AMP) shock. The SVAR model is associated with a more persistent effect of both shocks, but otherwise the IRFs are remarkably similar despite the fact that the shocks in the SVAR are estimated imposing only a limited number of identification restrictions.

[10]Intuitively, with policy rule (G.1) a shock to η_t^r that rises the interest rate by 25bp increases the surprise variable $\bar{S}_t$ roughly 4 times more than a shock to ϵ_t^r.

Figure G.2: Impulse Responses: VAR vs. SW model



Note. Impulse responses to a monetary policy shock (left column) and an anticipated monetary policy shock (right column). Red continuous line corresponds to the IRFs from the SW model, whereas blue dotted lines corresponds to the IRFs from the SVAR, the blue shaded area represents the 68% (point-wise) HPD credible sets for the IRFs.

H Comparison with Fed Staff Estimates in May 2006

In this section we compare results using our SVAR with internal exercises conducted by the Fed Staff using the FRB/US model and available as part of declassified historical materials. We have taken the results of the exercises published in the Bluebook dated May 5, 2006.[11] We re-estimate the model using the vintage of data available at that time. We also appended the unemployment rate to the dataset used in the VAR in order to obtain a conditional forecast of this variable as well that we can compare with the Fed staff estimates.

In this meeting, the FOMC was presented with two alternatives. The first is the Greenbook baseline in which the fed funds rate is raised to 5% and then lowered below 4.75%. To shed light on the implications of ending the process of policy firming, a second simulation considers a policy trajectory in which the federal funds rate is held constant at its current level through 2010 rather than being raised to 5%. Figure H.1 shows the comparison of the two scenarios, with the corresponding results published by the Fed staff. In the description of the Bluebook, the staff mentions that " monetary policy adjusts to hold core PCE inflation at about 2 percent" so we condition on inflation being at 2% in expectation at the end of the horizon. They also state the "assumption that this policy path is fully anticipated by financial market participants" which in our framework is operationalized as allowing both anticipated and unanticipated shocks to deviate from their unconditional distribution, and "the absence of any significant supply shocks or changes in expectations", which in our setting we model as all non-policy shocks to be at their unconditional distribution.

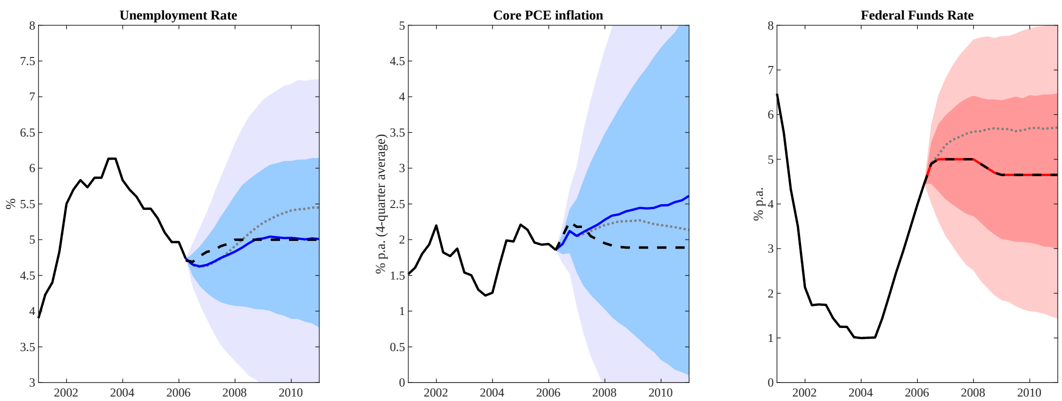
It is interesting to note that while the median of the SVAR result is quite close to the FRB/US result for the unemployment rate, inflation is projected to be higher with the SVAR than with the FRB/US in both scenarios. This is a result of the federal funds rate staying persistently below the SVAR unconditional forecast throughout the forecast horizon, reflecting a sequence of monetary policy shocks which also drive up inflation above the 2% target. One interpretation of our results is that the SVAR would have suggested that the Fed staff forecasts are too optimistic on the inflation front, and that further interest rate increases would be required to stabilize inflation. In the event, this is what happened, with the FOMC ultimately increasing the federal funds rate to 5.25% and inflation stabilizing around 2%. The mode of the KL is 0.80 for both exercises. The reason the

[11] See https://www.federalreserve.gov/monetarypolicy/files/FOMC20060510bluebook20060505.pdf (Chart 5).

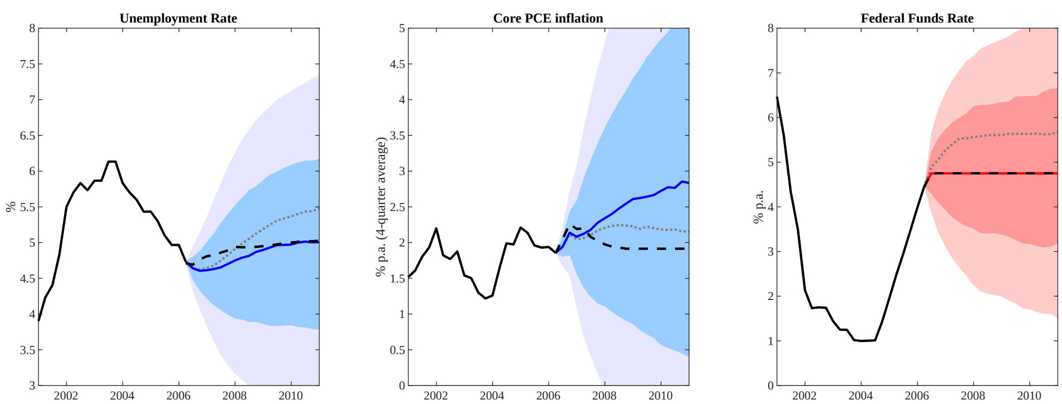
scenario is deemed to be a relatively implausible one is because it embodies a highly persistent deviation from the unconditional scenario at the end of the forecast horizon.

Figure H.1: FOMC Monetary Policy Alternatives: May 2006

(a) Baseline



(b) Lower near-term funds rate



Note: For each panel, the solid black lines represent actual data. In the third column, the solid red line and associated shaded areas are the median, 40 and 68% pointwise credible sets around the conditioning assumptions for the federal funds rate. In the first and second panel, the solid blue line and associated shaded areas are the median, 40 and 68% pointwise credible sets around the forecasts of the other variables given by the SVAR. The dotted gray lines represent the median unconditional forecast from the SVAR. The broken black lines represent instead the results of the simulations from the FRB/US model as reported in the Fed Bluebook in May 2006.

I Narrative Evidence of Fed forward guidance

In this appendix we provide evidence in support of the narrative identification restrictions used to complement the sign restrictions in the paper. In particular we report extract of FOMC documents in the periods 2003-2004 and 2011-2012 that justify our choices.

FOMC communication and forward guidance: 2003-2004

In order to enhance the communications with the public, since January 2000 the FOMC started to issue a brief public statement after every FOMC meeting. The FOMC's immediate announcement has contained a summary rationale for its policy decision and encapsulate the Committee's view of future developments. On some occasions, the announcement has directly hinted at the likelihood of changes in policy. The statement has been kept brief, partly to improve the predictability of the market reaction. Unsurprisingly, from the very beginning the FOMC statements have attracted considerable attention from the media and the markets, with the language closely watched with regard both to the current decision and to clues about future policy settings. In particular, special attention is paid to any changes in wording since the last statement. The first statements used the "balance of risk" assessment as a way of conveying the most likely 'tilt' in future policy. This practice had however at time come under intense scrutiny. In fact, "the meaning of the balance-of risks sentence is somewhat vague, giving rise to differences of opinion even among FOMC members and possibly leading to difficulties among outsiders in understanding its subtleties." (p. 191).[12]

Governor Bernanke highlights the importance of giving more precise guidance on the policy stance. In particular, for the August 12, 2003 meeting transcript Bernanke says (page 131):

"Our discussion yesterday and our experience of the past year have shown that effective communication is a big part of successful monetary policymaking. [. . .] We can communicate to financial markets how we expect to manage short-term interest rates in the future. If we then follow through on our commitments, the power of our interest rate instrument is multiplied many times over. [. . .] Nevertheless, communication that is entirely qualitative in nature also risks misunderstanding. [. . .] In an environment of low inflation and low interest rates, we need to seek ever greater clarity of communication to the markets and to the public." In mid-2003 the FOMC starts to give a more explicit reference to the future monetary policy path.

[12] See https://www.federalreserve.gov/monetarypolicy/files/FOMC20030624memo01.pdf

The September 14, 2003 FOMC meeting is dedicated to the discussion of the FOMC communication policy and procedure around it. In this meeting Gov. Bernanke makes the point that explicit communication in the FOMC statement should be used to convey information about the policy that is not only about the FOMC view on the economy: *"[. . .] we shouldn't be giving unconditional information about our interest rate setting. We should be trying to guide the market by providing conditional forecasts about which direction our policy may be going in light of our objectives, our views, or the nature of our rule."*

August 12, 2003 FOMC Meeting: Negative anticipated monetary policy shock

FOMC Statement *The Committee perceives that the upside and downside risks to the attainment of sustainable growth for the next few quarters are roughly equal. In contrast, the probability, though minor, of an unwelcome fall in inflation exceeds that of a rise in inflation from its already low level. The Committee judges that, on balance, the risk of inflation becoming undesirably low is likely to be the predominant concern for the foreseeable future. In these circumstances, the Committee believes that policy accommodation can be maintained for a considerable period.*

Minutes *While the Committee could not commit itself to a particular policy course over time, many of the members referred to the likelihood that the Committee would want to keep policy accommodative for a longer period than had been the practice in past periods of accelerating economic activity. Reasons for such an approach to policy stemmed from the need to encourage progress toward closing the economy's currently wide output gap and, with inflation already near the low end of what some members regarded as an acceptable range, to resist significant further disinflation. [. . .] At the same time, maintaining an accommodative policy stance was seen as involving little risk of inducing rising inflation so long as high levels of excess capacity and very competitive markets continued to characterize economic conditions.*

Additional support for the choice of this data as an example of forward guidance comes from the Transcripts of the 28 Jan 2004 FOMC meeting. For instance, in his prepared remarks the FOMC secretary Reinhart states: *"At times over the past six months, members have chaffed at the constraint imposed by the commitment to keep policy accommodative for a considerable period. But by being explicit to the public about this self-imposed constraint, you did help limit the tendency of market participants to build in unhelpfully aggressive expectations of policy firming, thereby keeping financial conditions accommodative at a time when you might have been concerned about the efficacy* *of alternative monetary policy actions. In the event, the expansion of aggregate demand did not falter, and there was no need to dig deeper into the toolkit of policymaking."*

In the same meeting Gov. Kohn makes the following remark: *"The "considerable period" phrase was inserted as a form of unconventional policy when we were concerned about deflation and the lower nominal bound."*

Jan. 28, 2004 FOMC Meeting: Negative anticipated monetary policy shock

The Committee perceives that the upside and downside risks to the attainment of sustainable growth for the next few quarters are roughly equal. The probability of an unwelcome fall in inflation has diminished in recent months and now appears almost equal to that of a rise in inflation. With inflation quite low and resource use slack, the Committee believes that it can be patient in removing its policy accommodation.

Minutes *"With regard to the wording of the Committee's press statement to be released shortly after the meeting, members discussed at some length the desirability of retaining a reference from earlier statements to the prospect that an accommodative policy could be maintained "for a considerable period." [...] All the members agreed that a change in wording was desirable, not to signal a policy tightening move in the near term, but rather to increase the Committee's flexibility to take such an action when it was deemed to be desirable and to underline that any such decision would be made on the basis of evolving economic conditions. [...] A number of members commented that expectations of sustained policy accommodation appeared to have contributed to valuations in financial markets that left little room for downside risks, and the change in wording might prompt those markets to adjust more appropriately to changing economic circumstances in the future. "*

A reading of the transcripts of the discussion in the FOMC highlights that the change in wording in the statement it transpires that the statement is meant to highlight the FOMC different assessment of state of the economy and therefore to align market expectations to the one of the FOMC, which sees a future tightening of policy more likely than what the markets were predicting.

Transcripts *" That there remains considerable mass on the possibility of earlier tightening embedded in financial market prices implies that there is a potential for the rally to be extended as expectations correct more to the Greenbook baseline. Why market participants might still expect earlier and more significant firming is evident in the bottom right panel: A survey of economists at eight dealers indicates that their outlook for inflation is decidedly less subdued than the staff's, even* *with a growth forecast that is less robust than the staff's. A distinct possibility is that many in the market have a gloomier view of the prospects for the growth of aggregate supply [...]*"

"In the Bluebook, we suggested adopting the notion of "patience" that the Chairman introduced in a recent speech, which market participants would probably take as implying that the Committee viewed events as such that it could be gradual in firming policy."

Chairman Greenspan speaking: *"[...] I think today is the day we should adjust our press statement and move to a reference to "patience." I think the downside risks to that change are small. I do think the market will react "negatively" as we used to say, but I'm not sure such a reaction would have negative implications, quite frankly. [...] If we go to "patience," we will have full flexibility to sit for a year or to move in a couple of months. [...] "*

May 4, 2004 FOMC Meeting: Positive anticipated monetary policy shock

"The Committee perceives the upside and downside risks to the attainment of sustainable growth for the next few quarters are roughly equal. Similarly, the risks to the goal of price stability have moved into balance. At this juncture, with inflation low and resource use slack, the Committee believes that policy accommodation can be removed at a pace that is likely to be measured."

The keyword in the statement, as evident from the minutes and traspits of the discussion, is "measured". This is meant to convey the intention of moving toward a tighter policy, but with at a pace that is slower than what market participants are expecting.

Minutes: *The Committee also discussed at length the advantages and disadvantages of modifying or dropping its statement in the announcement following the March meeting that "With inflation quite low and resource use slack, the Committee believes that it can be patient in removing its policy accommodation." All of the members agreed that, with policy tightening likely to begin sooner than previously expected, the reference to patience was no longer warranted. The Committee focused instead on a formulation that would emphasize that policy tightening, once it began, probably could proceed at a pace that would be "measured." A number of policymakers were concerned that such an assertion could unduly constrain future adjustments to the stance of policy should the evidence emerging in coming months suggest that an appreciable firming would be appropriate. Others, however, saw substantial benefits to inclusion of the proposed language. These members noted that current economic circumstances made it likely that the process of returning policy to a more neutral setting would be more gradual, once under way, than in past episodes when inflation was* *well above levels consistent with price stability.* *In addition, some policymakers observed that the timing and magnitude of future policy adjustments would ultimately be determined by the Committee's interpretation of the incoming data on the economy and prices rather than by its current expectation of those developments. On balance, all the members agreed that they could accept an indication in the statement that ". . . policy accommodation can be removed at a pace that is likely to be measured."*

Transcripts *In the Bluebook, we offered modified language that the "Committee believed that policy accommodation can likely be removed at a measured pace" to emphasize that you see yourselves entering a tightening phase but one that likely will not be as aggressive as in prior episodes. That emphasis seemed important because, as in the table at the bottom left, market participants currently anticipate about 200 basis points of firming in the next year. That is probably influenced by the experience of the three prior tightening phases, in which the funds rate moved an average of 2 2/3 percentage points higher within the first year. But with inflation lower now than in those prior episodes, you may see less need at this juncture for such firming, perhaps viewing as more likely something on the order of the 100 basis point increase assumed by the staff. In the Greenbook, the gradual realization by investors that you will not need to tighten as sharply as they now believe imparts an accommodative offset to your policy firming. Without that offset, you may be concerned that the swing of both monetary and fiscal policies toward more restraint may slow the growth of spending next year even more than is built into the Greenbook.*

FOMC Forward Guidance: 2011 - 2012

August 9, 2011 FOMC Meeting: Negative anticipated monetary policy shock

Statement. *"The Committee currently anticipates that economic conditions–including low rates of resource utilization and a subdued outlook for inflation over the medium run–are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013. [. . .] The Committee discussed the range of policy tools available to promote a stronger economic recovery in a context of price stability. It will continue to assess the economic outlook in light of incoming information and is prepared to employ these tools as appropriate."*

Minutes. *"In the discussion of monetary policy for the period ahead, most members agreed that the economic outlook had deteriorated by enough to warrant a Committee response at this meeting. While all felt that monetary policy could not completely address the various strains on the* *economy, most members thought that it could contribute importantly to better outcomes in terms of the Committee's dual mandate of maximum employment and price stability. In particular, some members expressed the view that additional accommodation was warranted because they expected the unemployment rate to remain well above, and inflation to be at or below, levels consistent with the Committee's mandate."*

From the transcripts it is clear that the statement chosen, over the various alternative considered reflects the intention of providing additional stimulus to the economy. This for instance transpires by the Secretary of the FOMC, Mr. English, description of the rationale for adopting the statement that was eventually released. Specifically, he mentions *"With the incoming data over the intermeeting period suggesting an even weaker outlook for economic activity, some participants may feel that it is now appropriate to provide additional accommodation, as in alternative A. [...] Regarding monetary policy, alternative A would include two steps to provide additional accommodation. First, paragraph 3 would provide more-explicit forward guidance about the expected path for the federal funds rate by specifying that exceptionally low levels were likely "at least through mid2013." Second, [...] Although expectations may have moved in recent days, market participants would probably still be surprised by the adoption of alternative A. Interest rates and the foreign exchange value of the dollar would likely fall, and stock prices would probably increase."*

January 24-25, 2012 FOMC Meeting: Negative anticipated monetary policy shock

Statement. *"To support a stronger economic recovery and to help ensure that inflation, over time, is at levels consistent with the dual mandate, the Committee expects to maintain a highly accommodative stance for monetary policy. In particular, the Committee decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that economic conditions–including low rates of resource utilization and a subdued outlook for inflation over the medium run–are likely to warrant exceptionally low levels for the federal funds rate at least through late 2014."*

Minutes. *"With respect to the statement to be released following the meeting, members agreed that only relatively small modifications to the first two paragraphs were needed to reflect the incoming information and the modest changes to the economic outlook implied by the recent data. In light of the economic outlook, almost all members agreed to indicate that the Committee expects to maintain a highly accommodative stance for monetary policy and currently anticipates that economic* *conditions–including low rates of resource utilization and a subdued outlook for inflation over the medium run–are likely to warrant exceptionally low levels for the federal funds rate at least through late 2014, longer than had been indicated in recent FOMC statements. In particular, several members said they anticipated that unemployment would still be well above their estimates of its longer-term normal rate, and inflation would be at or below the Committee's longer-run objective, in late 2014. It was noted that extending the horizon of the Committee's forward guidance would help provide more accommodative financial conditions by shifting downward investors' expectations regarding the future path of the target federal funds rate. Some members underscored the conditional nature of the Committee's forward guidance and noted that it would be subject to revision in response to significant changes in the economic outlook."*

From the transcripts it is clear that the statement chosen, over the various alternative considered reflects the intention of providing additional stimulus to the economy. This for instance transpires by the Secretary of the FOMC, Mr. English, description of the rationale for adopting the statement that was eventually released. Specifically, he mentions *"However, your SEP submissions suggest that you see only modest economic growth over coming quarters, with the unemployment rate declining even more slowly than you anticipated at the time of the November SEP. [...] With regard to inflation, the SEP results suggest that most of you see inflation as likely to be subdued. Against this backdrop, you may wish to ease financial conditions a bit further by providing forward guidance that pushes out the expected date of the liftoff of the federal funds rate. You may also think it helpful to provide clear guidance regarding the economic conditions that would warrant retaining the current very low level of the federal funds rate. [...] Market participants see significant odds that the Committee will raise the funds rate by the middle of 2014, and so specifying a date of "at least through late 2014" would likely push out somewhat expectations for the date of the first increase in the funds rate. Longer-term interest rates would likely decline a little, equity prices could rise some, and the foreign exchange value of the dollar might soften. "*

September 12-13, 2012 FOMC Meeting: Negative anticipated monetary policy shock

Statement. *" The Committee is concerned that, without further policy accommodation, economic growth might not be strong enough to generate sustained improvement in labor market conditions. Furthermore, strains in global financial markets continue to pose significant downside risks to the economic outlook. The Committee also anticipates that inflation over the medium term likely would* *run at or below its 2 percent objective.[...] To support continued progress toward maximum employment and price stability, the Committee expects that a highly accommodative stance of monetary policy will remain appropriate for a considerable time after the economic recovery strengthens. In particular, the Committee also decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015."*

Minutes. *"Members generally judged that without additional policy accommodation, economic growth might not be strong enough to generate sustained improvement in labor market conditions. [...] Inflation had been subdued, even though the prices of some key commodities had increased recently. Members generally continued to anticipate that, with longer-term inflation expectations stable and given the existing slack in resource utilization, inflation over the medium term would run at or below the Committee's longer-run objective of 2 percent.*

In their discussion of monetary policy for the period ahead, members generally expressed concerns about the slow pace of improvement in labor market conditions and all members but one agreed that the outlook for economic activity and inflation called for additional monetary accommodation. Members agreed that such accommodation should be provided through both a <u>strengthening of the forward guidance regarding the federal funds rate</u> and purchases of additional agency MBS at a pace of $40 billion per month. [...] With regard to the forward guidance, the Committee agreed on an extension through mid-2015, in conjunction with language in the statement indicating that it expects that a highly accommodative stance of policy will remain appropriate for a considerable time after the economic recovery strengthens. <u>That new language was meant to clarify that the maintenance of a very low federal funds rate over that period did not reflect an expectation that the economy would remain weak, but rather reflected the Committee's intention to support a stronger economic recovery.</u>"

The presentation material for this meeting associated with the alternative monetary policy options, presents simulations which highlight how the adopted statement was meant to provide additional stimulus to the economy. In fact, the secretary of the FOMC, Mr. English, states *"The black solid lines in the charts depict the experimental consensus forecast, which is conditioned on unchanged policy. The red dashed lines show the results of a more accommodative policy, one that's consistent with an alternative B that shifts back the date in the forward guidance to mid-2015 and, in addition to $30 billion of MBS purchases a month and completion of the MEP this year,* *involves buying longer-term securities at a rate of $75 billion a month through the middle of next year. [...]] So long as the public correctly anticipates that the Committee will follow this policy, the result is a more rapid economic recovery that takes the unemployment rate to 6.3 percent by the end of 2015, about 1/2 percentage point lower than in the consensus baseline; the inflation rate (shown at the bottom right) is a bit higher but remains near your 2 percent longer-run objective. [...] If, in reviewing the outlook under the unchanged policy, the Committee views the likely outcomes for employment and inflation as inconsistent with its mandate, it might choose to <u>ease policy by strengthening the forward guidance in the statement</u> and engaging in additional asset purchases. Alternative B, on page 5, may offer the Committee an attractive approach to moving strongly in that direction at this meeting, but without committing now to a large, discrete purchase program. "*

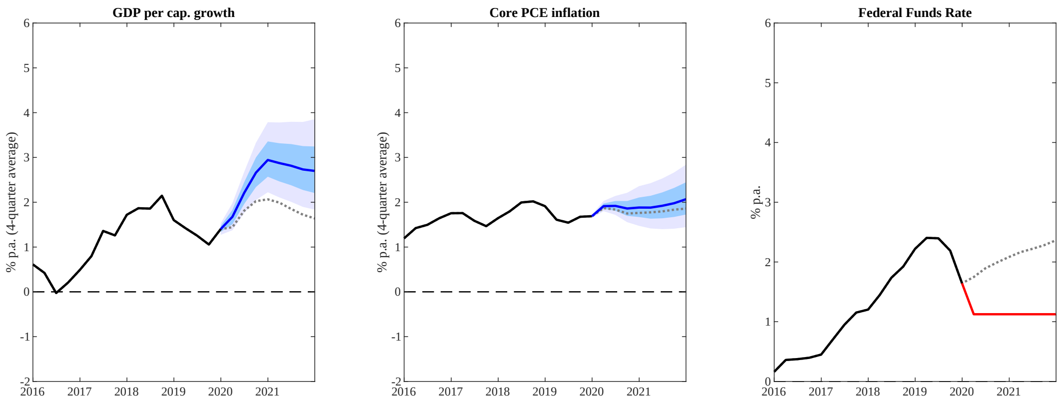
J Additional Figures

Figure J.1 analyzes the consequences of the alternative treatments of uncertainty for the density forecasts for the structural scenario: "What is the likely path of output and inflation, if monetary policy shocks lower the fed funds rate to 1% and keep it there for two years?"

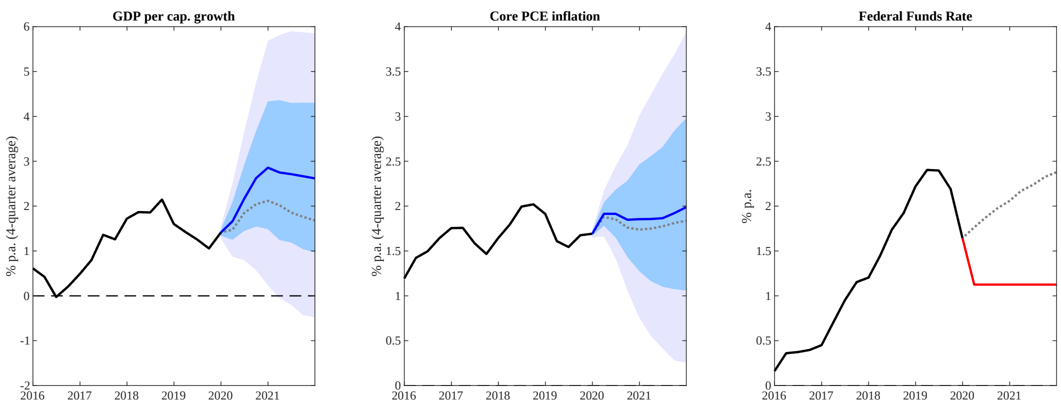
Figure J.2 shows an alternative scenario for the exercise of returning to the price level target using only unanticipated shocks. The results indicate that a more volatile interest rate path would be required to achieve the same objective. The fed funds rate would be cut to 1% by mid-2020 and then increased to 4.6% in 2023. During that year the probability of negative annual GDP growth would exceed 50%. Figure J.3 instead shows a scenario in which both anticipated and unanticipated shocks are used, but the price level returns to the target in 7 years instead of 4 years.

Figure J.1: Comparison under Different Treatments of Uncertainty

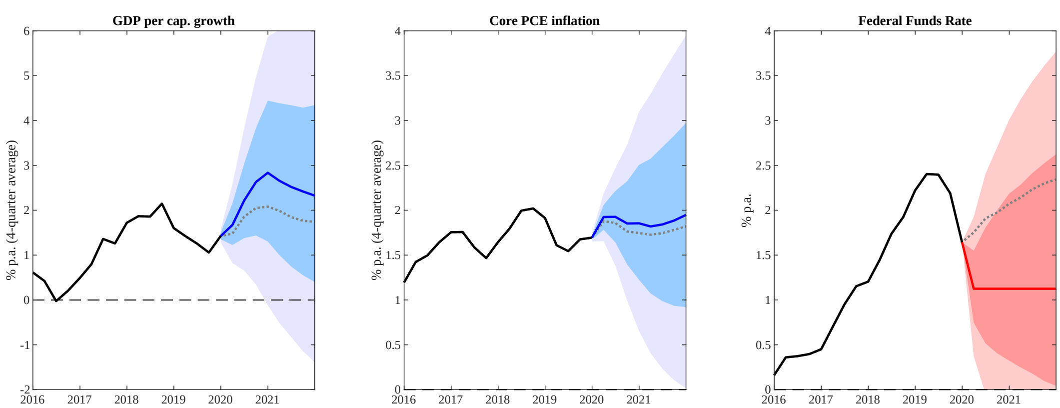
(a) No Uncertainty ($\boldsymbol{\Omega}_f = \mathbf{0}, \boldsymbol{\Omega}_g = \mathbf{0}$)



(b) Only Uncertainty about the Non-Driving Shocks ($\boldsymbol{\Omega}_f = \mathbf{0}, \boldsymbol{\Omega}_g = \mathbf{I}$)

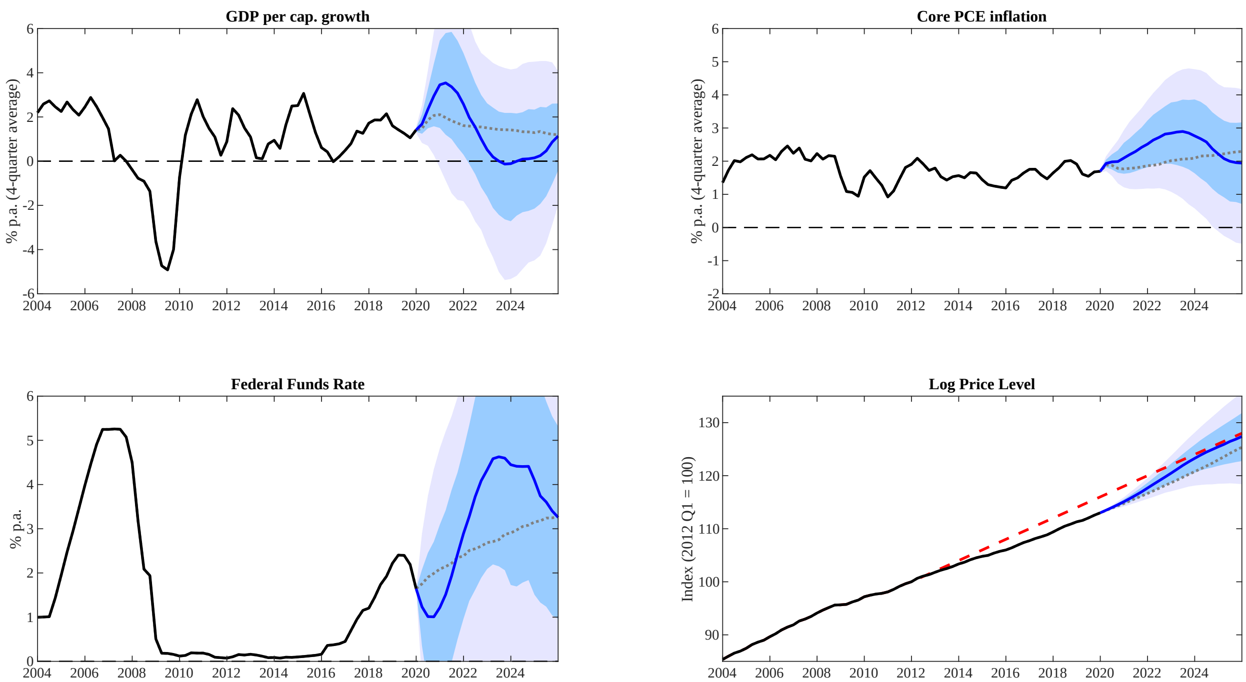


(c) Full Uncertainty ($\boldsymbol{\Omega}_f = \mathbf{D}\mathbf{D}', \boldsymbol{\Omega}_g = \mathbf{I}$)



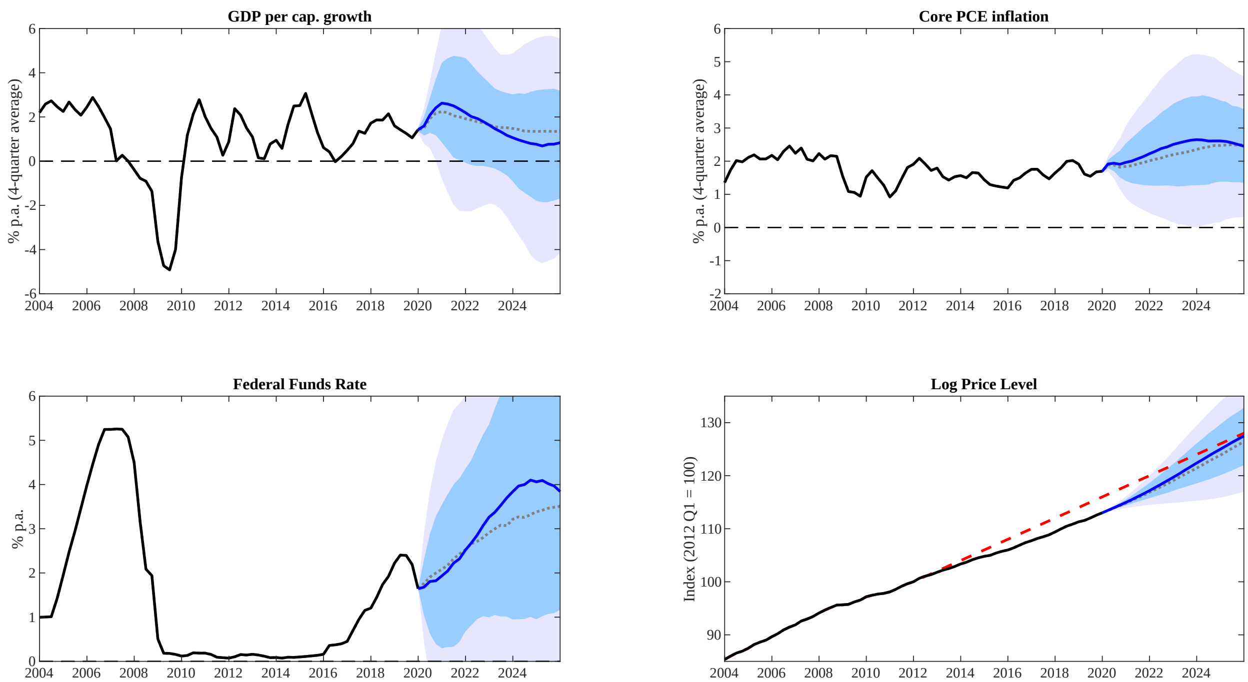
Note: For each panel, the solid black lines represent actual data, the solid red line is the conditioning assumption on the fed funds rate, the solid blue line is the median forecast for the remaining variables and periods, and the blue shaded areas denote the 40 and 68% pointwise credible sets around the forecasts. The dotted gray lines represent the median unconditional forecast.

Figure J.2: Returning to the Price Level Target: Unanticipated Shocks Only



Note: For each panel, the solid black lines represent actual data. The solid blue lines and associated shaded areas are the median, 40 and 68% pointwise credible sets around the forecasts of the other variables given such that the constraint that inflation returns to the price level target within a five-year horizon is satisfied. The dotted gray lines represent the median unconditional forecast.

Figure J.3: Returning to the Price Level Target: Seven Year Horizon



Note: For each panel, the solid black lines represent actual data. The solid blue lines and associated shaded areas are the median, 40 and 68% pointwise credible sets around the forecasts of the other variables given such that the constraint that inflation returns to the price level target within a seven-year horizon is satisfied. The dotted gray lines represent the median unconditional forecast.

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